

# Mathematical Models for Predicting Pollutant Transport in Soils at Decayed Waste Dumpsites of Gwari Market, Yola, Nigeria

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## Abstract

Due to the careless siting of human activities in Nigeria, resultant pollutants pass through porous soils to pollute the underlying groundwater resources. Consequently Darcy's Experimental set-up of Falling Head Permeability test was used to determine the velocities and travel distances of pollutants through soils leached from Gwari market waste dumpsites with a view to formulating mathematical equations for predicting the transport of pollutants through the porous soil media so as to safeguard the groundwater resources of the area. The dumpsite open year is (1999), 15 years old and spread on an area of 3.2 hectares with capacity of 60,000 tons. Results showed that the model advection velocity of 0.03845cm/sec. obtained is approximately equal to the observed laboratory velocity value determined as 0.03963cm/sec. Also, a model travel distance of 19.3cm obtained is approximately equal to an observed distance of 20cm determined in the laboratory. The analyses done implied that the models can be used to predict if pollutants from a dumpsite would travel to a river channel due to advection and dispersion processes within a specific given period which is a good pollution management strategy. The study recommended that in the planning and management of the physical environment, the knowledge of pollutants vertical velocity through porous soils should be integrated into water resources management for evaluating the effectiveness of pollutants mitigating alternatives, and in risk assessment to ensure environmental sustainability.

**Keywords:** mathematical models, pollutants, vegetable dumpsites, velocity, transport.

## 1. Introduction

Human activities have produced varieties of wastes that are normally discarded in low-lying areas without proper management strategies (ADSD, 2005). As such, a better way of evaluating the hydrodynamics of groundwater is to have a good knowledge of pollutants movement using numerical models which are major tools to predict the mobility of pollutants to and within soil system. Therefore, the spread of chemical pollutants in the subsurface and the final transport of these dissolved substances in soils have generated public interest on the quality of subsurface environment (Anand et al., 2004). Moreover, (Andreas, et al., 2000) asserted that an environmental issue that has achieved centre stage visibility is groundwater contamination and people are rightfully concerned about the siting of landfills/dumpsites near water resources because of its consequences. Besides, (Henry, 1856) observed that traditionally, most of the reported models in literature on pollutant transport through soils media is a function of physical, chemical and biological processes. The physical process involves advection and dispersion, the chemical process is usually a difficult one accompanied with complexation, while biological process could reduce contaminants through decay by microorganisms and so on (Garg, 2004).

Several research efforts have been made to control the widespread of pollutants in many parts of the world, including solid, liquid and gaseous that are still released into the environment on continuous basis. These resultant pollutants reach the groundwater system and contaminate them through the circulation of water (Keith and Richard, 1991). The mobility of these organic contaminants is governed by two key factors of sorption and chemical retardation (Ingebritsen and Sanford, 2014). The linear average velocity is responsible for the dispersion of contaminants in the aquifer, while sorption onto soil organic matter is responsible for the delay in contaminant migration (Fetter, 2013). The linear contaminant average velocity can be estimated rather accurately. However, the sorption capacity and chemical retardation of a particular soil cannot be precisely predicted and they vary from site to site (Lyman and Reehl, 2013). The aim of this paper is to construct mathematical models for predicting vertical movement of pollutants through porous soils at Gwari market decayed vegetable waste dumpsites at Yola with a view to establishing a reliable groundwater resources management strategy in the area and provide a simple methodology for numerical simulation of contaminants transport as a management strategy to protect these water resources. Part of this strategy is to construct mathematical models for predicting pollutants' movement through soils. When this is done, both vertical travel distance and travel time taken by a known pollutant can be calculated. The time for a known pollutant to travel to an aquifer of a known distance can be determined once their characteristics are known. In this way, the groundwater resources and people living around can be protected from ingesting pollutants. The following objectives were pursued to achieve this aim; (i) to carry out falling head test experiment on soils around the case study dumpsites, (ii) to determine velocities and travel distances of pollutants through the soil media, and (iii) to construct mathematical models for pollutants transport prediction.

## 2. Materials and Methods

### 2.1 Description of study sites:

The case study dumpsite (Fig. 1) is located at the north western part of Gwari sub-urban International Vegetables Market, established in 1999 in Jimeta-Yola town; the Headquarters of Adamawa State being one of the 36 states in the Federal Republic of Nigeria. The vegetables market lies between Latitudes  $9^{\circ}16'$  and  $9^{\circ}17'N$ ; Longitudes  $12^{\circ}00'$  and  $12^{\circ}26'E$  (UBRBDA, 2014). The market occupies an area of  $93,290.36m^2$  and the vegetable wastes dumpsite is 90m in length, width of 69m and depth of 6m with a capacity of 60,000tons spreading over an area of 3.2 hectares. Leafy and fruit vegetable wastes were co-disposed in the dumpsite with other items such as food wastes, plastic containers, motor parts, electronic parts, lubricating oil containers, etc. The landscape is low land with a slope of 0.033 and topography elevation of 500ft above the Mean Sea Level (White, 2010). Non point source (NPS) runoff drains through the decayed vegetable waste dumpsites and contaminate soils with heavy metals leached from these dumpsites before emptying into the adjacent River Benue about  $1\frac{1}{2}km$  away from these dumpsites.



Figure 1: Decayed Vegetable Waste Dumpsites at Gwari Market, Yola

### 2.2 Method of Testing Heavy Metal Contaminants and other Pollutants from Dumpsite Soils:

The main goal of this test was to analyse heavy metals distribution in the soils of the study area leached from these decayed waste dumpsites. The determination of heavy metals was done by applying the Aqua Regia extracting method where soil samples were collected from six points labelled alphabetically (A, B, C, D, E, F) according to how they were sampled from the field. This represents 2 meters at the upper part of dumpsites, directly under the dumpsites, and 2 meters at lower part of dumpsites. The samples were carefully air dried for 1 week and transferred to laboratory for grinding into fine particles using mortar and pestle. The samples were then passed through  $150\mu m$ ,  $63\mu m$ ,  $38\mu m$  sieves and collected at the bottom pan in a mechanical shaker after 30 minutes. Thereafter, 5g of each sample was weighed using digital weighing balance. 100ml of ultra pure water (upw) was added into clean conical flasks containing the samples. The solutions were digested for 3 hours using a digital orbital shaker set at 120rpm. Filtration of the samples was achieved by a vacuum pump. Filtrates from this pump were again passed through 0.22micros size of string filters. Working standards depending on the metals to be detected were prepared by withdrawing 1.0ml from Stock Standard (SS). Added to it approximately was, 1 - 2%  $HNO_3$  and top up to 100ml of upw. The samples were then taken to the Thermo Scientific iCE 3500 Atomic Absorption Spectrophotometer (AAS), for analyses after calibration of the machine was done for the reading of the toxic heavy metals in the samples. The heavy metals analysed for each soil sample were: Cd, Hg, Pb, Cu, Fe, Cr, Mn, Zn, As, Ni, Mg, Co).

### 2.3 Experimental set-up and Conceptual Frame Work:

This study is based on the experimental set-up of Fluid Flow rate. According to the fluid motion law, the rate of flow of groundwater through a column of saturated medium is:

- directly proportional to the difference in hydraulic head at the ends of the column, and
- inversely proportional to the length of column.

This experimental set-up is shown in (Fig.2). A metal cylinder of cross-sectional area A, was filled with unconsolidated soil samples collected from the case study dumpsites and made to be fitted at an angle,  $\theta$ , to the horizontal. The cylinder was made to have two hydraulic heads at the ends, in order to assume atmospheric pressure. The material was saturated with water by allowing it to flow through  $Q_1$ . The soil water containing dissolved nitrate ( $NO_3^-$ ) and Lead (Pb) contaminant compounds as detected from soil samples collected from the

dumpsites flowed out through  $Q_2$  then  $Q_1 = Q_2 = Q$

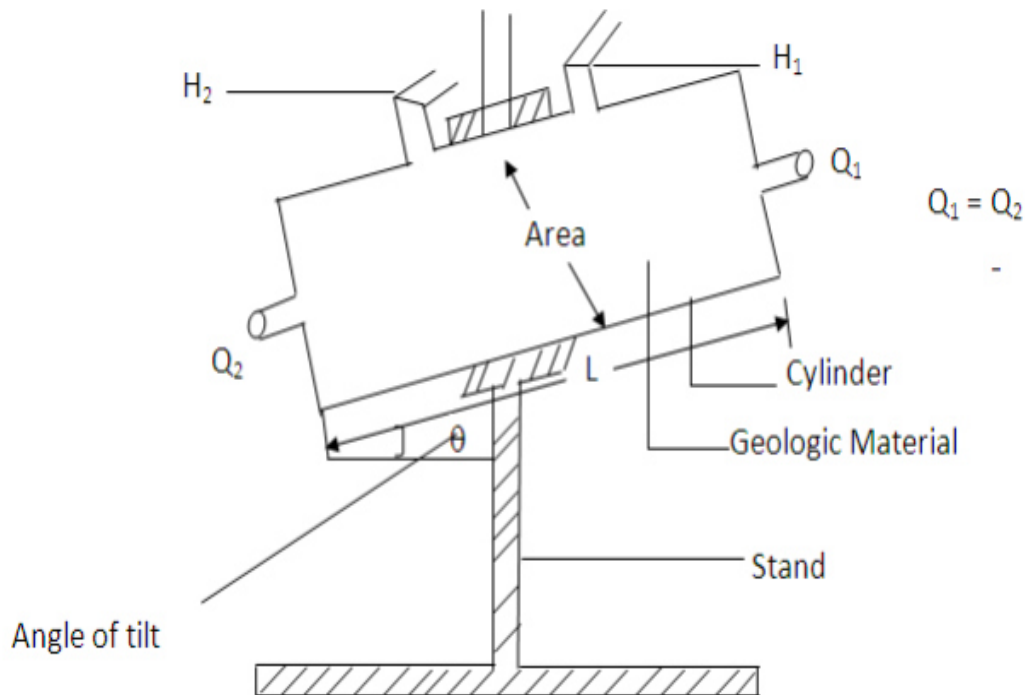


Fig.2: Laboratory Experimental Set-up

Putting fluid motion Law numerically, the model is computed according to (Lubo, et al., 2006) as follows:

$$V = \frac{h_1 - h_2}{KL} \quad (1)$$

Where;

$V$  = water velocity or specific discharge;

$K$  = hydraulic conductivity;

$h_1 - h_2$  = difference in Hydraulic head;

$L$  = length of the column.

Hydraulic gradient is the difference in hydraulic heads at two points, divided by the length,  $L$ , hence (1) becomes:

$$V = KL \quad (2)$$

According to Syed et al., (2008) in a pipe flow condition, which is analogous to flow in a porous medium (soil), the discharge,  $Q$ , is given by:

$$Q = VA \quad (3)$$

Where;

$A$  = the cross-sectional area of the cylinder;

$V$  = the velocity or specific discharge

From the above equation, the relationship of discharge to hydraulic conductivity was established as:

$$Q = KLA \quad (4)$$

This relationship is of fundamental importance in groundwater studies, particularly in a study of this nature in which the most common mode of contaminant migration in the subsurface is advective velocity in groundwater. Predicting the advective flow velocities based on the hydraulic gradient causing flow through soils of the decayed waste dumpsites is the main focus of the present study. This forms the basis for quantifying the rate of vertical fluid flow through unsaturated subsurface soil materials. This simple approach does not take into account of dispersion, diffusion, or adsorption of contaminants travel, which causes possible sorption and retardation of chemical species at groundwater flow circulated by advection. The types and sizes of porous medium as determined from the study site was sandy loam which was tested through particles size distribution and soil textural classification using Hydrometer Method. In this experiment,  $Q$  represents discharge, and it is expressed as discharge per unit time ( $m^3/s$ ).  $K$  is the hydraulic conductivity, and it indicates the quantity of water that flowed through a unit cross-sectional area  $A$ , per unit of time under a unit hydraulic gradient at a specified temperature. Here, interest is on vertical movement of pollutants; hence instead of tilting the cylinder to an angle  $\theta$  as in (Fig.2), the set-up is vertical (Fig.3).

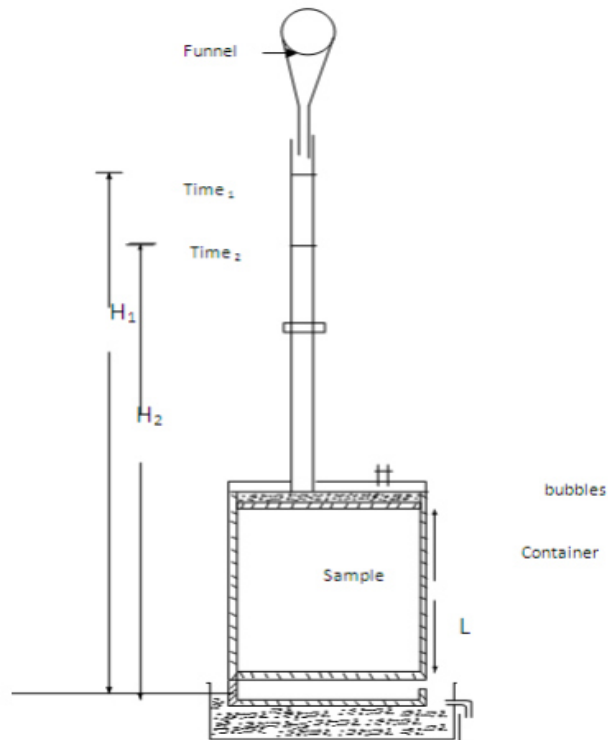


Fig. 3: Falling Head Permeameter Test Set-up

The experimental run was repeated five times and the average advection velocities with travel time were used in the model.

Construction of Mathematical models:

In order to study the pollutants velocity in porous soils, it is suitable to model the travel distance and travel time.

Thus, if  $\frac{dH}{dt}$  represents the rate of fall of head, the rate of flow is given by:  $(-)$   $a \cdot \frac{dH}{dt}$ , the negative sign means that the head falls with time. Equating this to the flow according to flow rate Law, we have:

$$-a \frac{dH}{dt} = k \cdot \left[ \frac{H}{L} \right] \cdot A \quad (5)$$

Where;

A = the inside area of the pipe (but in real life, will be the inside area of source of pollutants),

H = difference in fluid level (in real life, it is the hydraulic gradient),

L = length of the cylinder (in real life, it is the vertical thickness of the formation through which the fluid flows),

A = area of the cylinder (in real life, it is the surface covered by the fluid, i.e. the surface area through which it seeps to pollute the soils and groundwater),

K = the hydraulic conductivity of the soil sample (in real life, it is the hydraulic conductivity of the soil formation through which the pollutants flow),

i.e.

$$a \frac{dH}{H} = -k \cdot \frac{A}{L} \cdot dt \quad (6)$$

Integrating between  $H_1$ ,  $H_2$  and  $t_1$ ,  $t_2$ , we get,

$$a \int_{H_1}^{H_2} \frac{dH}{H} = -k \cdot \frac{A}{L} \cdot \int_{t_1}^{t_2} dt \quad (7)$$

$$a \cdot \ln H \Big|_{H_1}^{H_2} = (-) k \cdot \frac{A}{L} \cdot \left| t \right|_{t_1}^{t_2} \quad (8)$$

$$a \cdot (\ln H_2 - \ln H_1) = (-) k \cdot \frac{A}{L} \cdot (t_2 - t_1) \quad (9)$$

$$k = \frac{(-)aL \log_e \frac{H_2}{H_1}}{A(t_2 - t_1)} = a.L \frac{\log_e \frac{H_2}{H_1}}{A(t_2 - t_1)} \quad (10)$$

$$k = \frac{2.3aL \log_{10} \frac{H_1}{H_2}}{At} \quad (11)$$

$$k = \frac{2.3aL}{At} \text{Log}_{10} \frac{H_1}{H_2} \quad (12)$$

Making t the flow time, we have:

$$t = \frac{2.3aL \text{Log}_{10} \frac{H_1}{H_2}}{Ak} \quad (13)$$

But  $\text{Log}_{10} \frac{H_1}{H_2} = \text{hydraulic gradient}$ ; replacing  $\text{Log}_{10} \frac{H_1}{H_2}$  with i, we have:

$$t = \frac{2.3aLi}{AK} \quad (14)$$

But Velocity =  $\frac{\text{traveled distance}}{\text{time taken}}$

$$V = \frac{d}{t} \quad (15)$$

$$t = \frac{d}{v} \quad (16)$$

Equating eqns. (14 and 16);

$$\frac{d}{v} = \frac{2.3aLi}{AK} \quad (17)$$

If d = L;

Equation (17) becomes:

$$\frac{l}{v} = \frac{2.3ai}{AK} \quad (18)$$

$$v = \frac{AK}{2.3ai} \quad (19)$$

$$\text{Thus, } d = \frac{AKt}{2.3ai} \quad (20)$$

Equations (19 and 20) could be used to calculate the advection velocity and travel distance of pollutants when:

- i). the hydraulic gradient of the formation (soil) is known;
- ii). the hydraulic conductivity of the formation (soil) is known;
- iii). the travel time of the pollutants is known.

### 3. Results and Discussion

#### 3.1 Results of Target Pollutants from Waste Dumpsites used in the Experiment:

The results of heavy metal pollutants used in the laboratory experiment to predict the contaminants advection velocities and travel distances through porous soils on which decayed wastes were dumped is presented in Table 1.



Table 1: Results for Heavy Metal Concentrations, NO<sub>3</sub><sup>-</sup> and EC in Soil Samples

Parameters (mg/l):	Samples: A	B	C	D	E	F	WHO '07
Zinc, Zn	1057.12	180.09	1266.23	1316.17	729.40	798.07	150.00
Copper, Cu	187.13	44.70	255.90	756.90	248.53	113.46	200.00
Manganese, Mn	296.75	103.25	394.27	247.21	234.83	319.97	1,800
Iron, Fe	11104.55	5718.18	12127.3	10990.9	12195.5	14081.8	0.30
Cadmium, Cd	0.00	3.60	0.00	7.77	0.00	0.00	4.00
Lead, Pb	0.00	0.00	1071.5	0.00	0.00	0.00	200.00
Nickel, Ni	28.25	21.75	36.75	31.00	27.75	40.75	0.2 -10
Chromium, Cr	38.75	59.50	63.00	52.75	49.00	50.75	10.00
Cobalt, Co	23.00	20.25	20.00	19.75	18.25	24.25	25.00
Arsenic, As	Nd	Nd	Nd	Nd	Nd	Nd	95.00
Mercury, Hg	0.00	0.001	0.002	0.012	1.017	1.011	0.001
Magnesium	0.200	0.245	0.231	0.143	1.436	1.011	0.200
Nitrate, NO <sub>3</sub> <sup>-</sup>	0.025	0.021	0.028	0.014	0.018	0.028	50.00
Elect. Cond., EC (µs/cm)	90.00	250.00	280.00	80.00	110.00	350.00	250.00

*A – Soil sample collected at 2m from upper part of dumpsites, B – Soil sample from the beneath of decayed waste dumpsites, C – Soil sample collected at 2m from the lower part of dumpsites, D – Soil sample collected at 2m from the upper part of study dumpsite, E – Soil sample from beneath of decayed waste study dumpsites, F – Soil sample collected at 2m from the lower part of the case study dumpsite, and ND – Not detected.*

It has been observed from Table 1 above that the concentrations of most heavy metal pollutants were above the international requirements of threshold limits for WHO, 2007 soil quality standards. This indicates that soils of the study site are unhealthy for crop productions and even for provision of any human activity facilities because the environment could be lethal in the near future if these case study dumpsites are not evacuated.

### 3.2 Testing of the Two Formulae:

Taking parameter values from laboratory work, the accuracy of (19) can be confirmed as: Experimental results:  $Q = 1.2(\text{m}^3/\text{s})$ ;  $t = 518.16(\text{sec.})$ ;  $A = 180.75(\text{cm}^2)$ ;  $v = 0.039635(\text{cm/s})$ ;  $d = 20(\text{cm})$ ;  $K = 0.0054$ ;  $a = 12.52(\text{cm}^2)$ ; and hydraulic gradient at that point,  $i = 0.9542$ ; Chromium, Cr = 63.0 mg/l; and Nitrates, NO<sub>3</sub><sup>-</sup> = 0.028mg/l

Substituting these in the relationship below, then we have:

$$V = \frac{AK}{2.3ai} = \frac{180.75 * 0.0054}{2.3 * 12.52 * 0.9542} = 0.03845 \text{ cm/sec.}$$

The model advection velocity value of 0.03845(cm/sec) obtained is approximately equal to the observed laboratory velocity value obtained which was 0.03963(cm/sec.) for nitrate and chromium pollutants detected in the soil sample.

Similarly, substituting values from the measurements taken, the accuracy of (20) can also be confirmed as: Calculated  $A = 180.75(\text{cm}^2)$ ;  $K = 0.0054$ ;  $a = 12.52(\text{cm}^2)$ ; Travel time 518.16(sec.); and Hydraulic gradient at that point = 0.9542; Chromium, Cr = 63.0 mg/l; and Nitrates, NO<sub>3</sub><sup>-</sup> = 0.028mg/l. Substituting these values in the relationship below, then we have:

$$d = \frac{AKt}{2.3ai} = \frac{180.75 * 0.0054 * 518.16}{2.3 * 12.52 * 0.9542} = 19.3(\text{cm})$$

The model distance value of 19.3cm is approximately equal to the observed distance of 20cm obtained in the laboratory representing distance travelled by chromium and nitrate contaminants detected in the soil samples. Due to the complexity in the chemistry of the chemical species which can behave differently in the soil, this paper regrets as a limitation of the present study for its inability to predict possible sorption and retardation processes that govern contaminants transport mechanism of these heavy metals in the soil.

Therefore, the travel vertical distance of pollutants through a sedimentary formation (soil) can be calculated provided that the following input parameters are known:

- the cross-sectional area covered by the pollutants;
- the hydraulic conductivity of the formation through which the pollutants travel;
- the travel time;
- the cross-sectional area of the supplying medium pipe

v). the hydraulic gradient of the formation (soil) through which the pollutants travel.

#### 4. Conclusions

This study regrets that pollutant sorption onto soil grains and chemical retardation which are two key factors that normally governs pollutant mobility through porous soil media were not determined due to complexity of contaminant chemistry and movement through subsurface hence difficult to predict which indicates the limitation of the present study. However, the breakthrough of this analysis showed that two mathematical models useable to calculate the advective velocities and travel distances of pollutants, including:  $v = 0.0384(\text{cm/sec})$  and  $d = 19.3(\text{cm})$  were determined and results obtained were compared. The model results were slightly lower when compared to the experimental observed values. The comparison of the present findings with the available literature indicates that the results of the present study are in good agreement with the previous studies. For example, (Onwuka et al., 2012) Onwuemesi found that the modeled velocity and travel distance of contaminants through a rock formation were  $0.035(\text{cm/sec})$  and  $18(\text{cm})$  respectively while this study simulated velocity of  $0.0384\text{m/s}$  and travel distance of  $19.3\text{cm}$ .

The models can be used to predict if pollutants from a dumpsite would travel to a river channel due to advection and dispersion processes within a specific given period which is a good pollution management strategy to protect people and groundwater resources thus, ensuring environmental sustainability.

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